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BFV-BRST Quantization of the Proca Model based on the Batalin-Tyutin formalism

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ABSTRACT

We apply the Batalin-Tyutin Hamiltonian method to the abelian Proca model in order to convert a second class constraint system into a first class one systematically by introducing the new field. Then, according to the BFV formalism we obtain that the desired resulting Lagrangian preserving standard BRST symmetry naturally includes the well-known Stückelberg scalar related to the explicit gauge-breaking effect due to the presence of the mass term. Furthermore, we also discuss the nonlocal symmetry structure of this model in the context of the nonstandard BRST symmetry.

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1 Introduction

The Dirac method has been widely used in the Hamiltonian formalism [1] to quantize second class constraint system, which does not form a closed constraint algebra in Poisson brackets. However, since the resulting Dirac brackets are generally field-dependent and nonlocal, and have a serious ordering problem between field operators, these are under unfavorable circumstances in finding canonically conjugate pairs. On the other hand, the quantization of first class constraint system [2,3] has been well appreciated in a gauge invariant manner preserving Becci-Rouet-Stora-Tyutin (BRST) symmetry [4,5]. If second class constraint system can be converted into first class one in an extended phase space, we do not need to define Dirac brackets and then the remaining quantization program follows the method of Ref. [2-5]. This procedure has been extensively studied by Batalin, Fradkin, and Tyutin [6,7] in the canonical formalism, and applied to various models [8-10] obtaining the Wess-Zumino (WZ) action [11,12].

Recently, Banerjee [13] has applied the Batalin-Tyutin (BT) Hamiltonian method [7] to the second class constraint system of the abelian Chern-Simons (CS) field theory [14-16], which yields first class constraint algebra in an extended phase space by introducing new fields. As a result, he has obtained the new type of an abelian WZ action, which cannot be obtained in the usual path-integral framework. Very recently, we have quantized several interesting models [17] as well as the nonabelian CS case [18], which yield the weakly involutive first class system originating from the second class one, by generalizing this BT formalism [7,13]. As shown in these works, the nature of the second class constraint algebra originates from the symplectic structure of CS term, not due to the local gauge symmetry breaking. Banerjee and Ghosh [19] have also considered a massive Maxwell theory, which has the explicit gauge-breaking term, in the BT approach. However, all these analyses do not carry out the covariant gauge fixing procedure preserving the BRST symmetry. On the other hand, Lavelle and McMullan (LM) recently have found that QED exhibits a new nonlocal symmetry [20]. Several authors [21,22] have extensively studied following their works.

In the present paper, we shall apply the BT Hamiltonian method [7] to the Abelian

Proca theory revealing the Stückelberg effect [23]. In section 2, we apply this formalism to the abelian Maxwell (Proca) theory in order to systematically convert a second class constraint system into a first class one by introducing a new auxiliary field ρ . In section 3, we will briefly discuss the special unitary gauge fixing reproducing the original second class theory. In section 4, we show that by identifying this unphysical new field ρ with the Stückelberg scalar we naturally derive the Stückelberg scalar term related to the explicit gauge-breaking mass term through a standard BRST invariant gauge fixing procedure according to the Batalin-Fradkin-Vilkovisky (BFV) formalism. We also analyse the nonlocal symmetry structure, which exists in QED, of the Proca model in the context of the nonstandard BRST symmetry.

2 The BT Formalism

Now, we first apply the BT formalism to the abelian massive Maxwell theory in four dimensions, whose dynamics are given by

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu \right], \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $g_{\mu\nu} = \text{diag}(+, -, -, -)$.

The canonical momenta of gauge fields are given by

$$\begin{aligned} \pi_0 &= 0, \\ \pi_i &= \dot{A}^i + \partial_i A^0. \end{aligned} \quad (2)$$

Then, $\Omega_1 \equiv \pi_0$ is a primary constraint [1]. The canonical Hamiltonian is

$$H_c = \int d^3x \left[\frac{1}{2} \pi_i^2 + \frac{1}{4} F_{ij} F^{ij} + \frac{1}{2} m^2 ((A^0)^2 + (A^i)^2) - A_0 \Omega_2 \right], \quad (3)$$

where Ω_2 is the Gauss' law constraint, which comes from the time evolution of Ω_1 , defined by

$$\Omega_2 = \partial^i \pi_i + m^2 A^0. \quad (4)$$

The time evolution of the Gauss' law constraint generates no more independent constraints. As a result, the full constraints of this model are Ω_1 and Ω_2 . Then, they

consist of the second class constraint algebra as follows

$$\Delta_{ij}(x, y) \equiv \{\Omega_i(x), \Omega_j(y)\} = -m^2 \epsilon_{ij} \delta^3(x - y) \quad (i, j = 1, 2), \quad (5)$$

where we denote $\epsilon_{12} = \epsilon^{12} = 1$.

We now introduce new auxiliary fields Φ^i to convert the second class constraint Ω_i into first class one in the extended phase space, and assume that the Poisson algebra of the new fields is given by

$$\{\Phi^i(x), \Phi^j(y)\} = \omega^{ij}(x, y), \quad (6)$$

where ω^{ij} is an antisymmetric matrix. According to the BT method [7], the modified constraint in the extended phase space is given by the polynomials of the auxiliary fields Φ^i as follows

$$\tilde{\Omega}_i(\pi_\mu, A^\mu, \Phi^i) = \Omega_i + \sum_{n=1}^{\infty} \Omega_i^{(n)}; \quad \Omega_i^{(n)} \sim (\Phi^i)^n, \quad (7)$$

satisfying the boundary condition, $\tilde{\Omega}_i(\pi_\mu, A^\mu, \Phi^i = 0) = \Omega_i$. The first order correction term in the infinite series [7] is given by

$$\Omega_i^{(1)}(x) = \int d^3y X_{ij}(x, y) \Phi^j(y), \quad (8)$$

and the first class constraint algebra of $\tilde{\Omega}_i$ requires the condition as follows,

$$\Delta_{ij}(x, y) + \int d^3w d^3z X_{ik}(x, w) \omega^{kl}(w, z) X_{jl}(z, y) = 0. \quad (9)$$

As was emphasized in Ref. [13,17], there is a natural arbitrariness in choosing ω^{ij} and X_{ij} from Eq. (6) and Eq. (8), which corresponds to canonical transformation in the extended phase space [6,7]. Thus, without any loss of generality, we take the simple solutions as

$$\begin{aligned} \omega^{ij}(x, y) &= \epsilon^{ij} \delta^3(x - y), \\ X_{ij}(x, y) &= m \delta_{ij} \delta^3(x - y), \end{aligned} \quad (10)$$

which are compatible with Eq. (9) as it should be. Then, the modified constraint, $\tilde{\Omega}_i$ give a strongly first class constraint algebra,

$$\{\tilde{\Omega}_i(x), \tilde{\Omega}_j(y)\} = 0, \quad (11)$$

where

$$\tilde{\Omega}_i = \Omega_i + m\Phi^i \quad (12)$$

are the modified constraints including the auxiliary fields Φ^i in the extended phase space.

Next, we derive the corresponding involutive Hamiltonian in the extended phase space. It is given by the infinite series [7],

$$\tilde{H} = H_c + \sum_{n=1}^{\infty} H^{(n)}; \quad H^{(n)} \sim (\Phi^i)^n, \quad (13)$$

satisfying the initial condition, $\tilde{H}(\pi_\mu, A^\mu, \Phi^i = 0) = H_c$. The general solution [7] for the involution of \tilde{H} is given by

$$H^{(n)} = -\frac{1}{n} \int d^3x d^3y d^3z \Phi^i(x) \omega_{ij}(x, y) X^{jk}(y, z) G_k^{(n-1)}(z) \quad (n \geq 1), \quad (14)$$

where the generating functions $G_k^{(n)}$ are given by

$$\begin{aligned} G_i^{(0)} &= \{\Omega_i^{(0)}, H_c\}, \\ G_i^{(n)} &= \{\Omega_i^{(0)}, H^{(n)}\}_{\mathcal{O}} + \{\Omega_i^{(1)}, H^{(n-1)}\}_{\mathcal{O}} \quad (n \geq 1), \end{aligned} \quad (15)$$

where the symbol \mathcal{O} in Eq. (15) represents that the Poisson brackets are calculated among the original variables, i.e., $\mathcal{O} = (\pi_\mu, A^\mu)$. Here, ω_{ij} and X^{ij} are the inverse matrices of ω^{ij} and X_{ij} , respectively. Explicit calculations yield

$$\begin{aligned} G_1^{(0)} &= \Omega_2, \\ G_2^{(0)} &= m^2 \partial_i A^i, \end{aligned} \quad (16)$$

which are substituted in (14) to obtain $H^{(1)}$,

$$H^{(1)} = \int d^3x \left[m(\partial_i A^i) \Phi^1 - \frac{1}{m} (\partial^i \pi_i + m^2 A^0) \Phi^2 \right]. \quad (17)$$

This is inserted back in Eq. (15) in order to deduce $G_i^{(1)}$ as follows

$$\begin{aligned} G_1^{(1)} &= m\Phi^2, \\ G_2^{(1)} &= m\partial_i \partial^i \Phi^1. \end{aligned} \quad (18)$$

Then, we obtain $H^{(2)}$ by substituting $G_i^{(1)}$ in Eq. (14)

$$H^{(2)} = \int d^3x \left[-\frac{1}{2}(\Phi^2)^2 - \frac{1}{2}(\partial_i \Phi^1)(\partial^i \Phi^1) \right]. \quad (19)$$

Finally, since

$$G_i^{(n)} = 0 \quad (n \geq 2), \quad (20)$$

we obtain the complete form of the Hamiltonian \tilde{H} after the $n = 2$ finite truncations as follows

$$\tilde{H} = H_c + H^{(1)} + H^{(2)}, \quad (21)$$

which, by construction, is strongly involutive,

$$\{\tilde{\Omega}_i, \tilde{H}\} = 0. \quad (22)$$

This completes the operatorial conversion of the original second class system with Hamiltonian H_c and constraints Ω_i into the first class with Hamiltonian \tilde{H} and constraints $\tilde{\Omega}_i$. From Eqs. (11) and (22), one can easily see that the original second class constraint system is converted into the first class system if one introduces two fields, which are conjugated with each other in the extended phase space. Note that the origin of second class constraint is due to the explicit gauge symmetry breaking term in the action (1).

Next we consider the partition function of the model in order to present the Lagrangian corresponding to \tilde{H} in the canonical Hamiltonian formalism. As a result, we will unravel the correspondence of the Hamiltonian approach with the well-known Stückelberg's formalism. First, let us identify the new variables Φ^i as a canonically conjugate pair (ρ, π_ρ) in the Hamiltonian formalism, *i.e.*,

$$\Phi^i = (m\rho, \frac{1}{m}\pi_\rho) \quad (23)$$

satisfying Eqs. (6) and (10). Then, the starting phase space partition function is given by the Faddeev formula [3,24] as follows

$$Z = \int \mathcal{D}A^\mu \mathcal{D}\pi_\mu \mathcal{D}\rho \mathcal{D}\pi_\rho \prod_{i,j=1}^2 \delta(\tilde{\Omega}_i) \delta(\Gamma_j) \det | \{\tilde{\Omega}_i, \Gamma_j\} | e^{iS}, \quad (24)$$

where

$$S = \int d^4x \left(\pi_\mu \dot{A}^\mu + \pi_\rho \dot{\rho} - \tilde{\mathcal{H}} \right), \quad (25)$$

with Hamiltonian density $\tilde{\mathcal{H}}$ corresponding to Hamiltonian \tilde{H} (21), which is now expressed in terms of (ρ, π_ρ) instead of Φ^i . The gauge fixing conditions Γ_i are chosen so that the determinant occurring in the functional measure is nonvanishing. Moreover, Γ_i may be assumed to be independent of the momenta so that these are considered as Faddeev-Popov type gauge conditions [24].

Before performing the momentum integrations to obtain the partition function in the configuration space, it seems appropriate to comment on the involutive Hamiltonian. If we directly use the Hamiltonian (21) following the previous analysis done by Banerjee *et al.* [19], we will finally obtain the non-local action corresponding to this Hamiltonian due to the existence of $(\partial^i \pi_i)^2$ -term in the action when we carry out the functional integration over π_ρ later. Furthermore, if we use this Hamiltonian, we can not also naturally generate the first class Gauss' law constraint $\tilde{\Omega}_2$ from the time evolution of the primary constraint $\tilde{\Omega}_1$, which is the first class. Therefore, in order to avoid these serious problems, we use the equivalent first class Hamiltonian without any loss of generality, which only differs from the involutive Hamiltonian (21) by adding a term proportional to the first class constraint $\tilde{\Omega}_2$ as follows

$$\tilde{H}' = \tilde{H} + \frac{\pi_\rho}{m^2} \tilde{\Omega}_2. \quad (26)$$

Then, we have the natural first constraint system such that

$$\{\tilde{\Omega}_1, \tilde{H}'\} = \tilde{\Omega}_2, \quad \{\tilde{\Omega}_2, \tilde{H}'\} = 0. \quad (27)$$

Note that when we act this modified Hamiltonian (26) on physical states, the difference is trivial because such states are annihilated by the first class constraints. Similarly, the equations of motion for observable (*i.e.* gauge invariant variables) will also be unaffected by this difference since $\tilde{\Omega}_2$ can be regarded as the generator of the gauge transformations.

3 The Original Unitary Gauge Fixing

Now, we consider the following effective phase space partition function

$$\begin{aligned} Z &= \int \mathcal{D}\pi_\mu \mathcal{D}A^\mu \mathcal{D}\pi_\rho \mathcal{D}\rho \prod_{i,j=1}^2 \delta(\tilde{\Omega}_i) \delta(\Gamma_j) \det | \{ \tilde{\Omega}_i, \Gamma_j \} | e^{iS'}, \\ S' &= \int d^4x (\pi_\mu \dot{A}^\mu + \pi_\rho \dot{\rho} - \tilde{\mathcal{H}}'). \end{aligned} \quad (28)$$

The trivial π_0 integral is performed by exploiting the delta function $\delta(\tilde{\Omega}_1) = \delta(\pi_0 + m^2 \rho)$ in (28). On the other hand, the other delta function $\delta(\tilde{\Omega}_2) = \delta(\partial^i \pi_i + m^2 A^0 + \pi_\rho)$ can be expressed by its Fourier transform with Fourier variable ξ as follows

$$\delta(\tilde{\Omega}_2) = \int \mathcal{D}\xi e^{-i \int d^4x \xi \tilde{\Omega}_2}. \quad (29)$$

Making a change of variable $A^0 \rightarrow A^0 + \xi$, we obtain the action

$$\begin{aligned} S &= \int d^4x [\pi_i \dot{A}^i - m^2 \rho (\dot{A}^0 + \dot{\xi}) + \pi_\rho \dot{\rho} - \frac{1}{2} \pi_i^2 - \frac{1}{4} F_{ij} F^{ij} + \frac{1}{2} m^2 (A^0)^2 + \frac{1}{2} m^2 A_i A^i \\ &+ A^0 \partial^i \pi_i - m^2 \partial_i A^i \rho - \frac{1}{2m^2} \pi_\rho^2 + \frac{1}{2} m^2 \partial_i \rho \partial^i \rho - \xi \pi_\rho - \frac{1}{2} m^2 \xi^2], \end{aligned} \quad (30)$$

where the corresponding measure is given by

$$[\mathcal{D}\mu] = \mathcal{D}\xi \mathcal{D}\pi_i \mathcal{D}A^\mu \mathcal{D}\pi_\rho \mathcal{D}\rho \prod_j \{ \delta[\Gamma_j(A^0 + \xi, A^i, \pi_i, \rho, \pi_\rho)] \} \det | \{ \tilde{\Omega}_i, \Gamma_j \} |. \quad (31)$$

Performing the Gaussian integral over π_i , this yields the action as follows

$$\begin{aligned} S_u &= \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu + \pi_\rho (\dot{\rho} - \xi - \frac{1}{2m^2} \pi_\rho) \right. \\ &\quad \left. - m^2 \rho (\dot{A}^0 + \dot{\xi}) - m^2 \partial_i A^i \rho + \frac{1}{2} m^2 \partial_i \rho \partial^i \rho - \frac{1}{2} m^2 \xi^2 \right]. \end{aligned} \quad (32)$$

Now, we choose the unitary gauge as follows

$$\Gamma_i = (\rho, \pi_\rho). \quad (33)$$

Note that this gauge fixing is consistent because when we take the gauge fixing condition $\rho = 0$, another condition $\pi_\rho = 0$ is naturally generated from the time evolution of ρ , *i.e.*, $\dot{\rho} = \{ \rho, H_u \} = -\frac{1}{m^2} \pi_\rho = 0$, where the Hamiltonian H_u corresponds to the intermediate

action S_u . However, we can not smoothly choose the massless limit in this unitary gauge because $\dot{\rho}$ tends to infinity for this limit. In this gauge, we get

$$\{\tilde{\Omega}_i(x), \Gamma_j(y)\} = \epsilon_{ij} \delta^3(x - y). \quad (34)$$

Then, we easily recover the original system. Therefore, we can interpret the original system (1) as a gauge-fixed version of the extended gauge system (13), (21), and (26).

4 The BFV-BRST Gauge Fixing

In this section, we first briefly recapitulate the BFV formalism [2,3] which is applicable for the general theories with first-class constraints. For simplicity, this formalism is restricted to a finite number of phase space variables. This makes the discussion simpler and conclusions more apparent.

First of all, consider a phase space of canonical variables q^i, p_i ($i = 1, 2, \dots, n$) in terms of which the canonical Hamiltonian $H_c(q^i, p_i)$ and the constraints $\Omega_a(q^i, p_i) \approx 0$ ($a = 1, 2, \dots, m$) are given. We assume that the constraints satisfy the constraint algebra [2,3]

$$\begin{aligned} [\Omega_a, \Omega_b] &= i\Omega_c U_{ab}^c, \\ [H_c, \Omega_a] &= i\Omega_b V_a^b, \end{aligned} \quad (35)$$

where the structure coefficients U_{ab}^c and V_a^b are functions of the canonical variables. We also assume that the constraints are irreducible, which means that locally there exists an invertible change of variables such that Ω_a can be identified with the m -unphysical momenta.

In order to single out the physical variables, we can introduce the additional conditions $\Phi^a(q^i, p_i) \approx 0$ with $\det[\Phi^a, \Omega_b] \neq 0$ at least in the vicinity of the constraint surface $\Phi^a \approx 0$ and $\Omega_a \approx 0$. Then, Φ^a play the roles of gauge-fixing functions. That is to say, from the condition of the time stability of the constraints, there exists a family of phase space trajectories. By selecting one of these trajectories through the conditions of $\Phi^a \approx 0$, we can get the $2(n - m)$ dimensional physical phase space noted by q^* and p^* [1-3]. And then, $\Phi^a(q^i, p_i)$ can be identified with the m -unphysical coordinates.

The described dynamical system

$$\mathcal{Z} = \int [dq^i dp_i] \delta(\Omega_a) \delta(\Phi^b) \det | [\Phi^b, \Omega_a] | e^{i \int dx (p \dot{q} - H_c)} \quad (36)$$

is completely equivalent to an effective quantum theory only depending on the physical canonical variables q, p of the physical phase space [2,3]. Note that the constraints $\Omega_a \approx 0$ and $\Phi^a \approx 0$ together with the Hamilton equations may be obtained from a action

$$S = \int dt (p_i \dot{q}^i - H_c - N^a \Omega_a - B_a \Phi^a) , \quad (37)$$

where N^a and B_a are Lagrange multiplier fields canonically conjugated to each other, obeying the commutation relations

$$[N^a, B_b] = i \delta_b^a , \quad (38)$$

and the gauge-fixing conditions contain λ^a in the following general form

$$\Phi^a = \dot{N}^a + \chi^a(q^i, p_i, N^a) , \quad (39)$$

where χ^a are arbitrary functions. Furthermore, we can see that the Lagrange multiplier N^a become dynamically active, and B_a serve as their conjugate momenta. This consideration naturally leads to the canonical formalism in an extended phase space.

In order to make the equivalence to the initial theory with constraints in the reduced phase space, we may introduce two sets of canonically conjugate, anticommuting ghost coordinates and momenta $\mathcal{C}^a, \bar{\mathcal{P}}_a$ and $\mathcal{P}^a, \bar{\mathcal{C}}_a$ such that

$$[\mathcal{C}^a, \bar{\mathcal{P}}_b] = [\mathcal{P}^a, \bar{\mathcal{C}}_b] = i \delta_b^a . \quad (40)$$

The quantum theory is defined by the extended phase space functional integral

$$\mathcal{Z}_\Psi = \int [d\mu] e^{i S_\Psi} , \quad (41)$$

where the action is now

$$S_\Psi = \int dt \{ p_i \dot{q}^i + B_a \dot{N}^a + \bar{\mathcal{P}}_a \dot{\mathcal{C}}^a + \bar{\mathcal{C}}_a \dot{\mathcal{P}}^a - H_m + i[Q, \Psi] \} , \quad (42)$$

and $[d\mu]$ is the Liouville measure, i.e.,

$$[d\mu] = [dq^i dp_i dN^a dB_a d\mathcal{C}^a d\bar{\mathcal{P}}_a d\mathcal{P}^a d\bar{\mathcal{C}}_a] , \quad (43)$$

on the constraint phase space. Here, the BRST-charge Q and the fermionic gauge-fixing function Ψ are defined by

$$\begin{aligned} Q &= \mathcal{C}^a \Omega_a - \frac{1}{2} \mathcal{C}^b \mathcal{C}^c U_{cb}^a \bar{\mathcal{P}}_a + \mathcal{P}^a B_a, \\ \Psi &= \bar{\mathcal{C}}_a \chi^a + \bar{\mathcal{P}}_a N^a, \end{aligned} \quad (44)$$

respectively. H_m is the BRST invariant Hamiltonian, called the minimal Hamiltonian,

$$H_m = H_c + \bar{\mathcal{P}}_a V_b^a \mathcal{C}_b. \quad (45)$$

Now, in order to derive a BRST gauge-fixed covariant action for the abelian Proca model considered in the previous section, let us introduce the ghosts and anti-ghosts along with auxiliary fields $(\mathcal{C}^i, \bar{\mathcal{P}}_i)$, $(\mathcal{P}^i, \bar{\mathcal{C}}_i)$, (N^i, B_i) , where $i = 1, 2$, according to the above BFV formalism in the extended phase space. The nilpotent BRST-charge Q , the fermionic gauge-fixing function Ψ , and the minimal Hamiltonian H_m have the following concrete forms

$$\begin{aligned} Q &= \int d^3x [\mathcal{C}^i \tilde{\Omega}_i + \mathcal{P}^i B_i], \\ \Psi &= \int d^3x [\bar{\mathcal{C}}_i \chi^i + \bar{\mathcal{P}}_i N^i], \\ H_m &= \tilde{H}' - \int d^3x [\bar{\mathcal{P}}_2 \mathcal{C}^1], \end{aligned} \quad (46)$$

where

$$\chi^1 = A^0, \quad \chi^2 = \partial_i A^i + \frac{\alpha}{2} B_2, \quad (47)$$

as gauge fixing functions, and α is an arbitrary parameter. Note that the form of H_m is simpler than that in Ref.[8] due to our improved Hamiltonian (26) proposed in our previous works [17].

The BRST-charge Q , the fermionic gauge-fixing function Ψ , and the minimal Hamiltonian H_m satisfy the following relations,

$$\begin{aligned} i [Q, H_m] &= 0, \\ Q^2 &= [Q, Q] = 0, \\ [[\Psi, Q], Q] &= 0, \end{aligned} \quad (48)$$

where they are the conditions of physical subspace being imposed.

Then, the effective action is

$$S_{eff} = \int d^4x [\pi_0 \dot{A}^0 + \pi_i \dot{A}^i + \pi_\rho \dot{\rho} + B_2 \dot{N}^2 + \overline{\mathcal{P}}_i \dot{\mathcal{C}}^i + \overline{\mathcal{C}}_2 \dot{\mathcal{P}}^2] - H_{total}, \quad (49)$$

where $H_{total} = H_m - i[Q, \Psi]$. Note that we could suppress the term $\int d^4x (B_1 \dot{N}^1 + \overline{\mathcal{C}}_1 \dot{\mathcal{P}}^1) = -i[Q, \int d^4x \overline{\mathcal{C}}_1 \dot{N}^1]$ in the Legendre transformation by replacing χ^1 with $\chi^1 + \dot{N}^1$.

4.1 The Standard Local Gauge Fixing

The fields B_1 , N^1 , $\overline{\mathcal{C}}_1$, \mathcal{P}^1 , $\overline{\mathcal{P}}_1$, \mathcal{C}^1 , and A^0 are eliminated, and integration of π_0 gives the delta functional by using of Gaussian integration. Then we obtain the generating functional as follows

$$\mathcal{Z} = \int [\mathcal{D}\mu] \exp[iS_{eff}], \quad (50)$$

where the effective action is

$$\begin{aligned} S_{eff} = & \int d^4x [\pi_i \dot{A}^i + \pi_\rho \dot{\rho} + B \dot{N} + \overline{\mathcal{P}} \dot{\mathcal{C}} + \overline{\mathcal{C}} \dot{\mathcal{P}} - \frac{1}{2}(\pi^i)^2 - \frac{1}{2m^2}(\pi_\rho)^2 \\ & - \frac{1}{4}F_{ij}F^{ij} - \frac{1}{2}m^2(A^i)^2 - m^2\rho\partial_i A^i + \frac{1}{2}m^2\partial_i\rho\partial^i\rho \\ & + N(\partial_i\pi^i + \pi_\rho) + B(\partial_i A^i + \frac{1}{2}\alpha B) + \partial_i\overline{\mathcal{C}}\partial^i\mathcal{C} + \overline{\mathcal{P}}\mathcal{P}], \end{aligned} \quad (51)$$

and the Liouville measure of the extended phase space is given by

$$[\mathcal{D}\mu] = [\mathcal{D}\pi_i \mathcal{D}A^i \mathcal{D}\rho \mathcal{D}\pi_\rho \mathcal{D}B \mathcal{D}N \mathcal{D}\overline{\mathcal{P}} \mathcal{D}\mathcal{C} \mathcal{D}\overline{\mathcal{C}} \mathcal{D}\mathcal{P}]. \quad (52)$$

Here we have redefined $N^2 \equiv N$, $B_2 \equiv B$, $\overline{\mathcal{C}}_2 \equiv \overline{\mathcal{C}}$, $\mathcal{C}^2 \equiv \mathcal{C}$, $\overline{\mathcal{P}}_2 \equiv \overline{\mathcal{P}}$, and $\mathcal{P}^2 \equiv \mathcal{P}$. Performing the integrations of π^i , π_ρ , \mathcal{P} , and $\overline{\mathcal{P}}$, and identifying with $N = -A^0$, we get the following covariant effective action

$$S_{eff} = \int d^4x [-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2(A_\mu + \partial_\mu\rho)^2 - A^\mu\partial_\mu B + \frac{1}{2}\alpha(B)^2 - \partial_\mu\overline{\mathcal{C}}\partial^\mu\mathcal{C}], \quad (53)$$

which is invariant under the BRST transformation

$$\begin{aligned} \delta_B A_\mu &= -\lambda\partial_\mu\mathcal{C}, & \delta_B\rho &= \lambda\mathcal{C}, \\ \delta_B\mathcal{C} &= 0, & \delta_B\overline{\mathcal{C}} &= -\lambda B, & \delta_B B &= 0, \end{aligned} \quad (54)$$

where λ is a constant Grassmann parameter, and the corresponding final measure is given by

$$[\mathcal{D}\mu] = [\mathcal{D}A^\mu \mathcal{D}\rho \mathcal{D}B \mathcal{D}C \mathcal{D}\bar{\mathcal{C}}]. \quad (55)$$

Therefore, in Eq. (53) we see that the auxiliary BF field ρ is exactly the well-known Stückelberg scalar [23]. Note that in this gauge, we can smoothly choose the massless limit because the gauge fixing conditions (47) are independent of mass m^2 , and then obtain the well-known QED result. On the other hand, this BRST symmetry gives a conserved current as

$$J_{B\mu} = F_{\mu\nu}\partial^\nu\mathcal{C} + m^2(A_\mu + \partial_\mu\rho)\mathcal{C} + B\partial_\mu\mathcal{C}, \quad (56)$$

through Noether's theorem.

4.2 The Nonstandard Nonlocal Gauge Fixing

Consider the BFM formalism in the previous section up to the point, where the integration over the momentum π_ρ was performed and the following action was obtained.

$$S_{eff} = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2(A_\mu + \partial_\mu\rho)^2 - A^\mu\partial_\mu B + \frac{1}{2}\alpha B^2 - \partial_i\bar{\mathcal{C}}\partial^i\mathcal{C} + \bar{\mathcal{P}}\dot{\mathcal{C}} + \bar{\mathcal{C}}\dot{\mathcal{P}} + \bar{\mathcal{P}}\mathcal{P} \right]. \quad (57)$$

This action is invariant under the BRST transformation, which have the form

$$\begin{aligned} \delta_B A_0 &= -\lambda\mathcal{P}, & \delta_B A_i &= -\lambda\partial_i\mathcal{C}, & \delta_B \rho &= \lambda\mathcal{C}, \\ \delta_B \mathcal{C} &= 0, & \delta_B \bar{\mathcal{C}} &= -\lambda B, & \delta_B B &= 0, \\ \delta_B \mathcal{P} &= 0, & \delta_B \bar{\mathcal{P}} &= -\lambda[-\partial_i F^{0i} + m^2(\dot{\rho} + A^0)]. \end{aligned} \quad (58)$$

Now, if we perform the integration over the ghost fields instead of their momenta, we can also find the nonlocal symmetry structure in the Proca theory as well as QED [20]. First, performing the integration over the ghost field \mathcal{C} , we get the following delta function

$$\delta(\partial_i\partial^i\bar{\mathcal{C}} - \dot{\bar{\mathcal{P}}}) = \det(\partial_i\partial^i)\delta(\bar{\mathcal{C}} - \frac{1}{\partial_i\partial^i}\dot{\bar{\mathcal{P}}}). \quad (59)$$

Next, performing the integration over $\bar{\mathcal{C}}$, we get non-local ghost Lagrangian

$$S_{gh} = \int d^4x \left[\dot{\bar{\mathcal{P}}} \frac{1}{\partial_i \partial^i} \dot{\mathcal{P}} - \bar{\mathcal{P}} \mathcal{P} \right]. \quad (60)$$

Notice that the appearance of the nonlocal term in the ghost action has a result of this unusual integration. However, this form can be also simply obtained by the change of variables

$$\mathcal{C} \rightarrow \frac{1}{\partial_i \partial^i} \bar{\mathcal{P}}, \quad \bar{\mathcal{C}} \rightarrow \mathcal{P}, \quad (61)$$

in the Eq. (53). Under these replacements, we have the nonlocal BRST charge, that we call Q' , being

$$Q' = \int d^3x \left[B \bar{\mathcal{C}} + -\partial_i F^{0i} + m^2(\dot{\rho} + A^0) \frac{1}{\partial_i \partial^i} \bar{\mathcal{P}} \right]. \quad (62)$$

Then, the effective action is BRST invariant under the following transformations as

$$\begin{aligned} \delta'_B A_\mu &= -\lambda \partial_\mu \left(\frac{1}{\partial_i \partial^i} \bar{\mathcal{P}} \right), & \delta'_B \rho &= \lambda \frac{1}{\partial_i \partial^i} \bar{\mathcal{P}}, \\ \delta'_B \bar{\mathcal{P}} &= 0, & \delta'_B \mathcal{P} &= -\lambda B, & \delta'_B B &= 0. \end{aligned} \quad (63)$$

This nonlocal BRST symmetry yields a conserved current through Noether's theorem as follows

$$J'_{B\mu} = F_{\mu\nu} \partial^\nu \frac{1}{\partial_i \partial^i} \bar{\mathcal{P}} + m^2 (A_\mu + \partial_\mu \rho) \frac{1}{\partial_i \partial^i} \bar{\mathcal{P}} + B \partial_\mu \frac{1}{\partial_i \partial^i} \bar{\mathcal{P}}. \quad (64)$$

Note that performing the change of variable (61), these nonlocal symmetry and conserved current turn into just the original local theory (56).

In conclusion, we have applied our improved Batalin-Tyutin method [17], which systematically converts the second class system into the first class one by introducing the new auxiliary fields, to the Proca theory. According to the BFV formalism with the efficient first class Hamiltonian through BT analysis, we have shown that the resulting Proca Lagrangian preserving standard BRST symmetry naturally includes the well-known Stückelberg scalar needed for the anomaly free theory by identifying this scalar with one of auxiliary fields. Furthermore, we have shown that the nonlocal symmetry structure recently proposed in QED also exists in the Proca model through the nonstandard BRST gauge-fixing procedure.

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